On UV Finiteness of the Four Loop $\mathcal{N}$=8 Supergravity

Renata Kallosh

Department of Physics, Stanford University, Stanford, CA 94305

Abstract

The 4-loop 4-point amplitude in $\mathcal{N}$=8 d=4 supergravity is UV finite due to supersymmetry. Even better UV behavior of the 4-loop 4-point amplitude, analogous to that of $\mathcal{N}$=4 SYM theory, has been recently established by computation in [1]. All n-point 4-loop amplitudes with $n > 5$ are finite on dimensional grounds. However, the situation with the 5-point amplitudes remained unclear. In this paper we will show that the 5-point 4-loop amplitude must be finite due to $\mathcal{N}$=8 supersymmetry, despite the fact that R^5 has a supersymmetric generalization for $\mathcal{N}$=1, $\mathcal{N}$=2 and $\mathcal{N}$=4 SUSY. This means that all 4-loop amplitudes in $\mathcal{N}$= 8 supergravity are UV finite. We also discuss the current expectations for higher loops.

1 Introduction

Recent *tour de force* computations [1] of the 4-loop 4-point amplitude in $\mathcal{N}$=8 d=4 supergravity [2] points out towards the possibility of the all-loop UV finiteness of the theory. The purpose of this note is to clarify the supersymmetry predictions for the 4-loop $\mathcal{N}$=8 d=4 supergravity (SG) and comment on higher loop predictions.

The linearized 3-loop counterterm was constructed in [3], [4] and for a while it was considered as a candidate for a 3-loop logarithmic divergence. However, the computations in [5] have shown that the corresponding divergence is absent, in agreement with their earlier unitarity cut method expectations. Moreover, not only the term $\log\Lambda\, R^4$, but also $\frac{\partial^2 R^4}{\Lambda^2}$ and $\frac{\partial^4 R^4}{\Lambda^4}$ in d=4 were shown to cancel at the 3-loop level. This "superfiniteness" property still does not have a clear explanation, but it indicates that the formula for the critical dimension were the UV divergences start,

$$D_c = 4 + \frac{6}{L}\ , \tag{1.1}$$

may be valid in $\mathcal{N}$=8 SG. At the 3-loop level in $\mathcal{N}$=8 SG only the term

$$\kappa^4 \int d^4x\, \sqrt{-g}\, R^4 + ... \sim \kappa^4 \int d^4x\, \sqrt{-g}\, (R_{\alpha\beta\gamma\delta}\overline{R}_{\dot\alpha\dot\beta\dot\gamma\dot\delta})^2 + ... \tag{1.2}$$

could be associated with the logarithmic divergence in graviton amplitudes. Higher powers of curvature, which may have defined an independent higher-point amplitude divergence, are ruled out by dimensional considerations. Therefore, the computation of the 4-point amplitude in [5] was sufficient to establish the finiteness of all n-point amplitudes at the 3-loop level: The same counterterm responsible for the 4-point divergence (or its absence) is also responsible for the higher point divergence as it is simply a non-linear completion of the 4-point counterterm. Since the 4-point divergence is absent, all higher point amplitudes at 3 loops are also finite.

The situation with the 4-loop divergences requires a more detailed discussion. Even prior to the computation of Ref. [1] it was clear that there should not be any logarithmic divergences of the 4-loop 4-point amplitude. However, the authors found much more. They found that the superfiniteness in the 4-point amplitude takes place even at the 4-loop level. Thus the mysteries continued to accumulate, which gives an additional encouragement towards further investigation of the possible all-loop UV finiteness of $\mathcal{N}$=8 d=4 supergravity.

On the other hand, there are no calculations so far of the possible divergences of the 5-point amplitudes $\mathcal{N}$=8 d=4 supergravity, without which one cannot be sure of the full 4-loop finiteness of $\mathcal{N}$=8 d=4 supergravity. More exactly, the higher point counterterms at 4-loop order $\kappa^6 \int d^4x\, \sqrt{-g}\, R^n$ for $n > 5$ have positive dimension $2(n-5)$ and do not support logarithmic divergences. The only

remaining point to check is the 5-point graviton amplitude.[1] One may wonder whether the relevant counterterm $\kappa^6 \int d^4x \sqrt{-g}\, R^5 + ...$, which is not a non-linear completion of the 4-point counterterm, is available or forbidden by supersymmetry. We will start here with a review of the known facts on this in the literature.

The recent analysis of supersymmetric counterterms in [6] is based on the harmonic superspace construction in [7] (DHHK). It suggests that no UV divergences are to be expected at the 4-loop order. This includes the 4-point amplitudes as well as all other higher point amplitudes. Since it is not known whether the list of counterterms in harmonic superspace studied by DHHK in [7] includes all possible candidates for $\mathcal{N}$=8 supersymmetric counterterms[2] , one would like to have an independent information on existence/non-existence of $\mathcal{N}$=8 supersymmetrization of the R^5 term.

The computation of the 5-point 1-loop type II string amplitude was performed in [8] where it was shown that the R^5 term is absent. This, by itself, may not be sufficient to prove that the $\mathcal{N}$=8 SG in four dimensions will not have a 5-point 4-loop UV divergence, however, it makes it rather plausible. Moreover, the tree level computation of the 5-point graviton string amplitude was also performed [9] and it was shown that various contributions to the R^5 cancel. This tree level answer for the string amplitudes does not suffer from the problem of extra states of string theory versus $\mathcal{N}$=8 SG [15], which may affect the 1-loop computations of [8]. The fact of cancellation of the tree level R^5 term in string theory [9] is therefore, again, suggesting that $\mathcal{N}$=8 SG at the 4-loop level will not have a 5-point amplitude divergence. Still, the R^5 term could have been allowed by SUSY and just happen to have the coefficient zero at the tree and 1-loop level in string theory.

In view of all this indications that, most likely, R^5 does not have an $\mathcal{N}$=8 generalization, a direct $\mathcal{N}$=8 supersymmetry analysis is still desirable. If the R^5 is disallowed by supersymmetry, this means that the 5-point 4-loop amplitude is free of divergences due to $\mathcal{N}$=8 supersymmetric Ward identities. This is an unambigous prediction for computations which respect $\mathcal{N}$=8 supersymmetry. If supersymmetry forbids the R^5 terms, this makes the actual computation not necessary.

In this paper we will show that in $\mathcal{N}$=1, $\mathcal{N}$=2 and $\mathcal{N}$=4 SG theories one can construct linearized supersymmetric 5-point counterterms starting with R^5. It will be important therefore to study carefully what exactly is the situation in $\mathcal{N}$=8. For this purpose we will evaluate the existence of all possible supersymmetric invariants following the procedure developed in the past in [3], [4] for the 4-point case. We will present the suspects and rule them out case by case.

We will end this note by a short discussion of the possible directions of research of the UV properties of $\mathcal{N}$=8 SG.

[1] This issue was raised by A. Tseytlin. We are grateful to L. Dixon and Z. Bern who informed us about it.

[2] In what follows we will compare, with the help of P. Howe, our candidates with those studied by DHHK.

2 Analysis of d=4 4-loop supersymmetric candidate counterterms

In the 4-loop order no supersymmetric counterterm of the symbolic form $\kappa^6 \int d^4x \sqrt{-g}\, R^4\partial^2 + ...$ is available in d=4, therefore the absence of a logarithmic divergence in the 4-point amplitude is not surprising.

As a warm up consider the supersymmetrization of the 3-point 2-loop $\kappa^2 \int d^4xR^3$ and 5-point 4-loop graviton coupling $\kappa^6 \int d^4xR^5$ in $\mathcal{N}$=1 supergravity. We can use on shell a chiral conformal superfield $W_{\alpha\beta\gamma}$ of dimension 3/2 and its spinorial derivative $D_{(\delta}W_{\alpha\beta\gamma)} = R_{\alpha\beta\gamma\delta}$. For the 3-point amplitude at 2 loops in d=4 we may try

$$S_3 \sim \kappa^2 \int d^4x\, d^2\theta\, W_{\alpha\beta\gamma}\, W^{\gamma\xi\eta}\, W_{\xi\eta}{}^{\alpha} \ . \tag{2.1}$$

It is supersymmetric but has a wrong dimension, so we need an extra spinorial derivative insertion

$$S_3 \sim \kappa^2 \int d^4x\, d^2\theta\, W_{\alpha\beta\gamma}\, W^{\gamma\xi\eta}\, D^{\alpha}W^{\beta}{}_{\xi\eta} \ . \tag{2.2}$$

This term is not supersymmetric since the insertion of a spinorial derivative makes the superfield $D^{\alpha}W^{\beta}{}_{\xi\eta}$ non-chiral. This is a useful way to confirm the well known fact that R^3 does not have a supersymmetric partner even in $\mathcal{N}$=1 SG. The 5-point amplitude at 4 loops, however, has an $\mathcal{N}$=1 supersymmetric version, namely

$$S_5^{N=1} \sim \kappa^6 \int d^4x\, d^2\theta d^2\bar{\theta}\, W_{\alpha\beta\gamma}\, W^{\gamma\xi\eta}\, D^{\alpha}W^{\beta}{}_{\xi\eta}\overline{W}_{\dot\alpha\dot\beta\dot\gamma}\overline{W}^{\dot\alpha\dot\beta\dot\gamma} + h.c. \tag{2.3}$$

It corresponds to the following combination of the curvature spinors

$$\kappa^6 \int d^4x\, R_{\alpha\beta\gamma\delta}\, R^{\gamma\delta\xi\eta}\, R_{\xi\eta}{}^{\alpha\beta}\overline{R}_{\dot\alpha\dot\beta\dot\gamma\dot\delta}\overline{R}^{\dot\alpha\dot\beta\dot\gamma\dot\delta} + h.c. \tag{2.4}$$

In $\mathcal{N}$=2 supergravity the linearized superfield of dimension 1 is $W_{\alpha\beta}$, which starts with the vector field strength spinor $F_{\alpha\beta}$. The 5-point supersymmetric generalization of the R^5 term (2.4) is

$$S_5^{N=2} \sim \kappa^6 \int d^4x\, d^4\theta d^4\bar{\theta}\, W_{\alpha\beta}\, D^{\beta}W^{\gamma\delta}\, D^{\gamma}W^{\delta}{}_{\alpha}\overline{W}_{\dot\alpha\dot\beta}\, \overline{W}^{\dot\alpha\dot\beta} + h.c. \tag{2.5}$$

At the level of $\mathcal{N}$=4 supergravity there is a dimension zero chiral superfield W and the generalization of the R^5 term (2.4) is

$$S_5^{N=4} \sim \kappa^6 \int d^4x\, d^8\theta d^8\bar{\theta}\, \epsilon_{ijkl}\, D^i_{\alpha}D^j_{\beta}W\, D^{\alpha k}W\, D^{\beta l}W\, \overline{W}\,\overline{W} + h.c. \tag{2.6}$$

What is available in $\mathcal{N}$=8 case? The full superspace integrals $\kappa^6 \int d^4xd^{32}\theta\mathcal{L}(W, D, \partial)$ depending on the linearized dimensionless superfield W_{ijkl} and its spinorial and space-time derivatives have positive mass dimension > 6 and will not supply the relevant supersymmetric invariant. We will study here the actions over the subspaces of the 32 θ's.

Thus we would like to look carefully for the counterterms, candidate for the 4-loop 5-point amplitudes, which are not related to a non-linear completion of the 4-point counterterms, and make sure that all possibilities are taken into account. We will use here the same method [3], [4] which in the past allowed us not to miss the 3-loop 4-point candidate counterterm. Now we will apply this method to the 4-loop 5-point case.

We are looking for the linearized supersymmetric version of $\kappa^6 \int d^4x \sqrt{-g}\, R^5 + ...$ The linearized superfield of $\mathcal{N}$=8 supergravity is $W_{ijkl} = \frac{1}{4!}\epsilon_{ijklmnpr}\overline{W}^{mnpr}$. We will use here, for simplicity, the setting of Ref. [3] where the linear superfield W_{1234} depends only on 16 θ's

$$W \equiv W_{1234} = \overline{W}^{5678} \equiv W(x', \theta_B) \ , \qquad \theta_B = (\theta_1, \theta_2, \theta_3, \theta_4; \bar\theta^5, \bar\theta^6, \bar\theta^7, \bar\theta^8) \tag{2.7}$$

in a special basis defined in [3], $x'_{\alpha\dot\alpha} = x_{\alpha\dot\alpha} + i\sum_1^4 \theta_i \sigma_{\alpha\dot\alpha}\bar\theta^i - i\sum_5^8 \bar\theta^j \sigma_{\alpha\dot\alpha}\theta_j$. The 3-loop 4-point candidate counterterm is

$$S^{L=3} = \kappa^4 \int d^4x d^{16}\theta_B W^4 \sim \kappa^4 \int d^4x \sqrt{-g}\, (R_{\alpha\beta\gamma\delta}\overline{R}_{\dot\alpha\dot\beta\dot\gamma\dot\delta})^2 + ... \tag{2.8}$$

Since W_{1234}^4 depends only on 16 θ_B, this expression is supersymmetric. Each superfield has a graviton spinor $R_{\alpha\beta\gamma\delta}$ (or $\overline{R}_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$) with 4 θ's (or 4 $\bar\theta$'s). Therefore one of the terms, a 4-graviton part, is invariant under $SU(8)$, so the supersymmetric partners are also $SU(8)$ invariant. A manifestly $SU(8)$ form of this 3-loop counterterm was constructed in [4] using the representations theory of $SU(8)$ and the Yang tableaux.

Now we would like to increase the power of κ by 2 to describe the 4-loop counterterm.

$$S_4^{L=4} = \kappa^6 \int d^4x d^{16}\theta_B W^4 \partial^2 \ . \tag{2.9}$$

Here $W^4\partial^2$ is a symbolic expression which means that two space time derivatives are inserted between 4 superfields W^4. In fact, the action is symmetric in 4 superfields, in the Fourier space we would have

$$S_4^{L=4} \sim \delta^4(p_1+p_2+p_3+p_4) W(p_1)W(p_2)W(p_3)W(p_4)(s+t+u) \ . \tag{2.10}$$

Since in the 4-point amplitude $s+t+u=0$, there is no 4-loop counterterm supporting the logarithmic divergence. This explains why the 4-point amplitude at 4-loop order is finite by supersymmetry.

For the 5-point amplitude we will first identify the supersymmetric invariants and afterwards check their $SU(8)$ invariance. The first indication of the $SU(8)$ invariance will be the presence of the 5-graviton term (2.4).

On dimensional grounds with $S_5^{L=4} = \kappa^6 \int d^4x d^{2m}\theta \mathcal{L}(W, D, \partial)$ we see that $m = 10 - \dim \mathcal{L}$ where $\dim \mathcal{L} \geq 0$. This means that we have to check the case of 16, 18 and 20 θ-integration with $\mathcal{L}(W, D, \partial)$

depending on the linearized dimensionless superfield W_{ijkl} and its spinorial and space-time derivatives. There is no way to have less than 16 θ-integration since each W_{ijkl} depends at least on 16 θ's.

The first attempt is[3]

$$S_5^{L=4} = \kappa^6 \int d^4x d^{16}\theta_B W^5 \partial^2 \ , \tag{2.11}$$

where ∂^2 means that two space-time derivatives are inserted between 5 superfields in an arbitrary way. It looks supersymmetric, since the integrand depends only on 16 θ_B. However, the gravity part of W has 4 θ or 4 $\bar\theta$, so this expression does not have the 5-graviton part which would be neutral in $SU(8)$. It has, for example, a square of the Bel-Robinson tensor times a scalar field with specific choice of $SU(8)$ indices, in our case ϕ_{1234}, which clearly violates $SU(8)$.

Second attempt[4] is to replace ∂^2 by 4 fermionic derivatives D_α, which hit some of the superfields, or to replace one ∂ by 2 fermionic derivatives. This has the correct dimension and may have a 5-graviton term, for example:

$$S_5^{L=4} = \kappa^6 \int d^4x d^{16}\theta_B W^5 D_\theta^4 \ . \tag{2.12}$$

However, when we hit the superfield $W = W_{1234}$ by a spinorial derivative, say D^4_α, it becomes a linearized superfield with the first component equal to a spinorial field, a **56** of $SU(8)$, namely $\chi_{\alpha 123}$. Consider the properties of this superfield $\chi_{ijk\,\beta}$, which under supersymmetry transforms into the vector field strength $F_{\alpha\beta\, ij}$ and into the derivative of the scalar $P_{\dot\alpha\beta[ijkl]}$

$$D^k_\alpha \chi_{ijk\,\beta} = F_{\alpha\beta\, ij} \ , \qquad D_{\dot\alpha l}\chi_{ijk\,\beta} = P_{\dot\alpha\beta[ijkl]} \ . \tag{2.13}$$

This means that the spinorial superfield $\chi_{123\,\beta}$ still depends on $(\theta_1, \theta_2, \theta_3; \bar\theta^4, \bar\theta^5, \bar\theta^6, \bar\theta^7, \bar\theta^8)$. However, it does not depend on θ_4 anymore, instead it depends on $\bar\theta^4$. The remaining scalar superfields W_{1234}, which are not hit by the spinorial derivatives (as we have only 4), still depend on the original combination of θ's, but each of the χ fields has some of the Grassmann variables switched partially to the new ones. The integral in eq. (2.12) is therefore not supersymmetric.

The next case is

$$S_5^{L=4} = \kappa^6 \int d^4x d^{16}\theta_B d^2\bar\theta^4 W^3_{1234}\chi_{\alpha 123}\chi^\alpha_{123} + h.c. \tag{2.14}$$

This expression looks supersymmetric since the Lagrangian depends on all 18 fermionic directions. However, it is possible to perform the integration over $d^2\bar\theta^4$ since W^3_{1234} does not depend on these fermionic directions. Each of these derivatives will hit only one of the spinorial superfields and produce $\partial_{\alpha\dot\beta}W_{1234}$. The expression becomes equivalent to the one in eq. (2.11) and is, therefore, ruled out.

---

[3]Such term was considered in DHHK in [7] and ruled out, P. Howe, private communication

[4] This term contains both the W and χ fields which obey different constraints. It has not been studied in an explicit DHHK analysis in [7], but can be shown to be not supersymmetric, in agreement with our argument below, P. Howe, private communication.

In case of 20 fermionic integrations, dimension does now permit any spinorial derivative insertions and the W^5_{1234} terms depends only on 16 θ's, the integral vanishes, there is no counterterm. We may also try to have a Lagrangian depending on W^3_{1234} and two other superfields depending on some of θ_B as well as 4 other θ's. For example W_{1235} depend on $\theta_1, \theta_2, \theta_3, \theta_5; \bar\theta^4, \bar\theta^6, \bar\theta^7, \bar\theta^8$

$$S_5^{L=4} = \kappa^6 \int d^4x d^{16}\theta_B d^2\bar\theta^4 d^2\theta_5 W^3_{1234} W^2_{1235} \ . \tag{2.15}$$

This looks supersymmetric, but the 5-graviton term is not there as one can check looking at each superfield θ^4 and $\bar\theta^4$ terms [5]. There are no other sub-superspace integrals depending on any combination of the superfields of the theory with any insertion of superspace derivatives, which in principle may serve as 5-point 4-loop counterterms.

Thus we conclude that all possibilities to construct the 5-point 4-loop candidate counterterm failed, the amplitude must be finite for the reason of supersymmetry and dimension.

3 Discussion

In this paper we have directly established that there is no $\mathcal{N}$=8 supersymmetric generalization on the $\kappa^6 \int d^4x\, (R_{....})^5$ counterterm. This is in complete agreement with other indications of the same fact, coming from [6] - [9]. As the result, there is no need to compute the 5-point 4-loop amplitude in $\mathcal{N}$=8 SG.

What is in the future for $\mathcal{N}$=8 SG now that 4-loop amplitudes are established to be finite and even superfinite according to eq. (1.1)? It was pointed out in [10], [11] that the UV properties of $\mathcal{N}$=8 SG may be studied in the light-cone unconstrained superspace [12] which admits a set of Feynman rules with one scalar superfield. Only physical degrees of freedom are propagating in this unitary gauge where all local symmetries are fixed. The counterterms for generic L-loop divergences have not been constructed yet in the light-cone formalism. They are known to exist [13], [3] starting from 8-loop order in terms of the Lorentz covariant on-shell geometric superfields. However, they may or may not lead to UV divergences. We have seen repeatedly in computations in [5] and [1] that the unexplained cancellations may take place.

The analysis performed in [11] shows that the relevant linearized counterterms are non-local in the light-cone formalism, which may explain the finiteness of d=4 theory before L=7. When $E_{7(7)}$ symmetry is added to the light-cone analysis, it may lead to the proof of an all loop finiteness of perturbative $\mathcal{N}$=8 SG. We have shown in [11] that a better understanding of the structure of the Feynman graphs of the light-cone $\mathcal{N}$=8 SG may be useful and may lead to conclusive statements on

[5]This type of an invariant was studied in DHHK [7] and ruled out, P. Howe, private communication.

the puzzling UV properties of the theory. Other proposals suggesting a possibility of UV finiteness of $\mathcal{N}$=8 SG [14], [15], [16], [17] will likely be clarified and developed in view of the recent impressive computations in [1]. Hopefully the UV status of perturbative $\mathcal{N}$=8 SG will be established.

Acknowledgments

I thank Z. Bern, M. Green, L. Dixon, P. Howe, K. Stelle, S. Stieberger, A. Tseytlin and P. Vanhove for the discussion of the 5-point amplitudes. This work is supported by the NSF grant 0756174.

References

[1] Z. Bern, J. J. Carrasco, L. J. Dixon, H. Johansson and R. Roiban, "The Ultraviolet Behavior of N=8 Supergravity at Four Loops," arXiv:0905.2326 [hep-th].

[2] E. Cremmer and B. Julia, "The SO(8) Supergravity," Nucl. Phys. B **159**, 141 (1979); B. de Wit and H. Nicolai, "N=8 Supergravity," Nucl. Phys. B **208**, 323 (1982); B. de Wit and D. Z. Freedman, "On SO(8) Extended Supergravity," Nucl. Phys. B **130**, 105 (1977); B. de Wit, "Properties Of SO(8) Extended Supergravity," Nucl. Phys. B **158**, 189 (1979).

[3] R. E. Kallosh, "Counterterms in extended supergravities," Phys. Lett. B **99** (1981) 122;

[4] P. S. Howe, K. S. Stelle and P. K. Townsend, "Superactions," Nucl. Phys. B **191**, 445 (1981).

[5] Z. Bern, J. J. Carrasco, L. J. Dixon, H. Johansson, D. A. Kosower and R. Roiban, "Three-Loop Superfiniteness of $\mathcal{N}$=8 Supergravity," Phys. Rev. Lett. **98**, 161303 (2007) [arXiv:hep-th/0702112]; Z. Bern, J. J. M. Carrasco, L. J. Dixon, H. Johansson and R. Roiban, "Manifest Ultraviolet Behavior for the Three-Loop Four-Point Amplitude of N=8 Supergravity," Phys. Rev. D **78**, 105019 (2008) [arXiv:0808.4112 [hep-th]].

[6] G. Bossard, P. S. Howe and K. S. Stelle, "The ultra-violet question in maximally supersymmetric field theories," arXiv:0901.4661 [hep-th].

[7] J. M. Drummond, P. J. Heslop, P. S. Howe and S. F. Kerstan, "Integral invariants in N = 4 SYM and the effective action for coincident D-branes," JHEP **0308**, 016 (2003) [arXiv:hep-th/0305202].

[8] D. M. Richards, "The One-Loop Five-Graviton Amplitude and the Effective Action," JHEP **0810**, 042 (2008) [arXiv:0807.2421 [hep-th]].

[9] L. Dixon, S. Stieberger, private communication.

[10] R. Kallosh, "On a possibility of a UV finite N=8 supergravity," arXiv:0808.2310 [hep-th].

[11] R. Kallosh, "N=8 Supergravity on the Light Cone," arXiv:0903.4630 [hep-th].

[12] L. Brink, O. Lindgren and B. E. W. Nilsson, "N=4 Yang-Mills Theory On The Light Cone," Nucl. Phys. B **212**, 401 (1983); A. K. H. Bengtsson, I. Bengtsson and L. Brink, "Cubic Interaction Terms For Arbitrarily Extended Supermultiplets," Nucl. Phys. B **227**, 41 (1983). S. Ananth, L. Brink and P. Ramond, "Eleven-dimensional supergravity in light-cone superspace," JHEP **0505**, 003 (2005) [arXiv:hep-th/0501079]. L. Brink, S. S. Kim and P. Ramond, "$E_7(7)$ on the Light Cone," JHEP **0806**, 034 (2008) [arXiv:0801.2993 [hep-th]].

[13] P. S. Howe and U. Lindstrom, "Higher Order Invariants In Extended Supergravity," Nucl. Phys. B **181**, 487 (1981).

[14] N. Berkovits, "New higher-derivative R^4 theorems," Phys. Rev. Lett. **98**, 211601 (2007) [arXiv:hep-th/0609006]; M. B. Green, J. G. Russo and P. Vanhove, "Non-renormalisation conditions in type II string theory and maximal supergravity," JHEP **0702**, 099 (2007) [arXiv:hep-th/0610299]; M. B. Green, J. G. Russo and P. Vanhove, "Ultraviolet properties of maximal supergravity," Phys. Rev. Lett. **98**, 131602 (2007) [arXiv:hep-th/0611273].

[15] M. B. Green, H. Ooguri and J. H. Schwarz, "Decoupling Supergravity from the Superstring," Phys. Rev. Lett. **99**, 041601 (2007) [arXiv:0704.0777 [hep-th]].

[16] H. J. Schnitzer, "Reggeization of N = 8 supergravity and N = 4 Yang-Mills theory," arXiv:hep-th/0701217.

[17] N. Arkani-Hamed, F. Cachazo and J. Kaplan, "What is the Simplest Quantum Field Theory?," arXiv:0808.1446 [hep-th].